Superconducting single-electron transistor and the ϕ-modulation of supercurrent

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An analytical expression for the supercurrent of a superconducting single-electron transistor (SSET) is derived. The derivation is based on analogy between the model Hamiltonian for $E_{\rm J} > E_{\rm C}$ and a discrete, one-dimensional harmonic oscillator (1DDHO). The resulting supercurrent is nearly identical to the supercurrent obtained from a continuous harmonic oscillator Hamiltonian.

The superconducting single-electron transistor consists of two consequent Josephson junctions and an intervening island on which the amount of charge can be controlled by a gate voltage. The relevant energy scales of the system are given by the Josephson energy $E_{\rm J}$ and the charging energy $E_{\rm C} := (-2e)^2/[2(C_1+C_2+C_{\rm g})]$ where C_1, C_2 and $C_{\rm g}$ are the capacitances of the two junctions and the gate capacitance, respectively. For independent junctions the Hamiltonian of the system is given by

$$H = H_{\rm C} - \sum_{j=1}^{2} E_{\rm J,j}\cos(\phi_j), \qquad (1)$$

where $H_{\rm C}$ gives the charging energy of the island and ϕ_j is the phase difference across the $j^{\rm th}$ junction.

The proper variables for description of the system are the phase difference across the array $\phi = \phi_1 + \phi_2$, and the number of Cooper pairs on the island N.[1] The phase difference ϕ is a constant of motion if the voltage across the SSET is ideally biased to zero. The Hamiltonian is fixed using the arguments given in Ref. 2, i.e. by taking $C_j = c_j C$ and $E_{{\rm J},j} = c_j E_{\rm J}$, where $c_1^{-1} + c_2^{-1} = 2$. The normalised gate charge $q = V_{\rm g}C_{\rm g}/(-2e)$ sets the amount of free charge on the island to $(-2e)(N-q)$. In the charge state representation the Hamiltonian reads[3,4]

$$\begin{aligned} H_{\rm C} &= E_{\rm C}\sum_N (N-q)^2 |N,\phi\rangle\langle N,\phi|, & (2)\\ H_{\rm J} &= -(E_{\rm J}/2)(c_1^2+c_2^2+2c_1c_2\cos(\phi))^{1/2} \\ &\quad \times \sum_N \left(e^{i\theta(\phi)}|N+1,\phi\rangle\langle N,\phi| + \text{h.c.}\right), & (3)\end{aligned}$$

where $\tan(\theta) = (c_1-c_2)\tan(\phi/2)/(c_1+c_2)$. In the limit of vanishing charging energy when $H_{\rm C}\to 0$, the ground state energy and supercurrent are given by[4]

$$\begin{aligned} E(\phi) &= -E_{\rm J}(c_1^2+c_2^2+2c_1c_2\cos(\phi))^{1/2}, & (4)\\ I_{\rm S}^{(0)} &= \frac{-2e}{\hbar}\frac{\partial E(\phi)}{\partial\phi} = \frac{(-2e/\hbar)E_{\rm J}c_1c_2\sin(\phi)}{(c_1^2+c_2^2+2c_1c_2\cos(\phi))^{1/2}}. & (5)\end{aligned}$$

If the charging effects are not negligible the Hamiltonian $H_{\rm C} + H_{\rm J}$ expressed in unit of $E_{\rm C}$ is identical to that of a 1DDHO with coupling constant $\varepsilon_\phi := E(\phi)/E_{\rm C}$. The eigenenergies are independent of the phase factor $e^{i\theta(\phi)}$ which simply fixes the relative phase between consequtive charge states $|N\rangle$ and $|N+1\rangle$.

For a continuous HO with the same ε_ϕ the eigenenergies are given by $E_j = -\varepsilon_\phi + \sqrt{2\varepsilon_\phi}(j+\frac{1}{2})$. In case of a 1DDHO with large ε_ϕ the bottom of the well is lifted by approximately $\frac{1}{8}$ and oscillator frequency $\sqrt{2a}$ is replaced by $\sqrt{2a}-\frac{1}{8}$. With these modifications numerically obtained eigenstates satisfy the virial theorem $\langle H_{\rm J}\rangle = \langle H_{\rm C}\rangle$ quite well. The agreement is best for the ground state for which the expression

$$E_0(\varepsilon_\phi) = -\varepsilon_\phi + \sqrt{\varepsilon_\phi/2} + 1/16 \qquad (6)$$

very accurate for $\varepsilon_\phi > 10$ and even at $\varepsilon_\phi \approx 2$ the error is smaller than 0.01 for any q. Because of the constant correction the derivative $\partial E_0/\partial\varepsilon_\phi$ is the same as in the continuous case. For weaker couplings with $\varepsilon_\phi \lesssim 2$ the minimum position q of the potential energy becomes important, but direct diagonalisation of the Hamiltonian is simple. When Eq. (6) is valid we obtain the final result

$$I_{\rm S}^{\rm SSET}(\phi) = I_{\rm S}^{(0)}[1-(8\varepsilon_\phi)^{-1/2}], \qquad (7)$$

where $I_{\rm S}^{(0)}$ is the supercurrent in the absence of charging effects. The magnitude of the correction $(8\varepsilon_\phi)^{-1/2}$ is of the order of 10 % when $\varepsilon_\phi \sim 10$. The correction slightly decreases the maximal obtainable supercurrent and it is important for nearly homogeneous arrays ($c_1 \approx 1$) as the coupling strength E_ϕ becomes small near $\phi = (2k+1)\pi$, where k is an integer.

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